Dresher’s Guessing Game

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Drescher’s Guessing Game

Alice thinks of a number between 1 and N

Bob tries to repeatedly guess that number

Alice tells him whether the guess is too high or too low or correct

Bob pays Alice one dollar for each guess
Selmer Johnson:

Case $n = 11$. Here $V(11) = 3 \frac{1}{52}$.

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<tr>
<th>Blue</th>
<th>58 29 29 34 25 22 25 34 29 29 58</th>
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<tbody>
<tr>
<td>Red</td>
<td>$S_1$ 3 4 2 3 4 1 4 3 4 2 3</td>
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<td>$S_2$ 3 4 2 3 4 1 4 3 2 4 3</td>
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<td>$S_4$ 3 2 3 1 4 5 3 4 2 4 3</td>
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<td>$S_5$ 3 2 3 1 4 3 4 2 4 3 4</td>
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<td>$S_6$ 3 2 3 1 4 3 4 2 5 4 3</td>
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58 29 29 34 25 22
54 6 6 6 6 8 1 1122
6 6 8 4 6 8 1 1122
87 6 6 6 7 4 4 1122
18 6 6 5 5 7 5 1122
6 7 5 7 3 8 3 1122
15 6 6 8 3 8 3 1122

The computation appears to get more difficult for $n = 12$ and beyond. Nevertheless, considerable reduction in the list of Red strategies is accomplished by the techniques of this paper.
Probability Distribution Alice
N=11
First 11 values of the game

N=2 to 11, observe that the numerator increases at N=11

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<td>11</td>
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Group Russian Roulette

In a room stand $n$ armed and angry people. At each chime of a clock, everyone simultaneously spins around and shoots a random other person. The persons shot fall dead and the survivors spin and shoot again at the next chime; eventually, either everyone is dead or there is a single survivor.

As $n$ grows, what is the limiting probability that there will be a survivor?

Peter Winkler:
Mathematical Puzzles
Solution: Amazingly, this probability does not tend to a limit; as $n$ grows, the probability varies subtly, but relentlessly, according to the fractional part of the natural logarithm of $n$. (For a related result, see H. Prodinger, “How to Select a Loser,” *Discrete Math* **120** (1993) pp. 149–159.)
Ulam’s Guessing Game

Alice thinks of a number between 1 and 100
Bob tries to repeatedly guess that number
Now Alice is allowed to lie once or twice
Bob pays Alice one dollar for each guess

The literature on this game is extensive
Randomize binary trees

Select a **fixed** binary tree, say on 6 numbers; select a **random** subset, say \{1,3,5,6,9,10\}; guess the remaining numbers \{2,4,7,8\} later
Randomize binary trees

Select a **fixed** binary tree, say on 6 numbers; select a **random** subset, say \{1,3,5,6,9,10\}; guess the remaining numbers \{2,4,7,8\} later
Asymptotic value of the game

Lemma

Let $N = 2^k - 2^d$ and let $y \leq N$ be any natural number. Then

$$V(N + y) \leq k - \frac{N - y}{N + y} + \frac{N}{N + y} \cdot \frac{1}{2^d}$$

If $N$ goes to infinity, this is close to the value of the game if Alice plays uniformly.
Asymptotic solution

**Theorem:**
If $N$ goes to infinity, the value of Dresher’s guessing game is equal to the game in which Alice plays uniformly.
Numerical solution of the game

Problem for linear programming: large matrix

Bob has many strategies, Alice has few strategies
Numerical solution of the game

Problem for linear programming: large matrix

Bob has many strategies, Alice has few strategies

Computing the optimal search tree against a particular strategy of Alice is a standard dynamic programming problem

Solution of the problem: do not store Bob’s strategies, construct the best by dynamic programming
Dynamic pivot part in the linear program

\[ a(1) \ a(2) \ a(3) \ldots \ a(N) \]

\[ \begin{array}{cccc}
  \ b(1) \\
  \ b(2) \\
   \cdot \\
   \cdot \\
  \ b(N) \\
\end{array} \]

BASE

compute optimal tree against \( a(i) \) and put it in the base
Approximation value game
N=20
Approximation value game
N=100
Runtime compared to N
Number of simplex iterations
Value of Game
N=253

3342819507234448752049668779569347604847528010229
divided by

474649422880958300359976770147691113271801061922
Probability Distribution Alice
N=11
Probability Distribution Alice
N=124
Probability Distribution Alice
N=200
Probability Distribution Alice
N=253
Thank you for your attention!